Risk Metrics and Coherence

Michael Schilmoeller

for the

Resource Adequacy Technical Committee

Friday, October 7, 2005

Northwest Power and Conservation Council

Overview

- Issues with risk measures
- Coherent measures of risk
- Relevance to resource adequacy and the metrics the Resource Adequacy Technical Committee is discussing

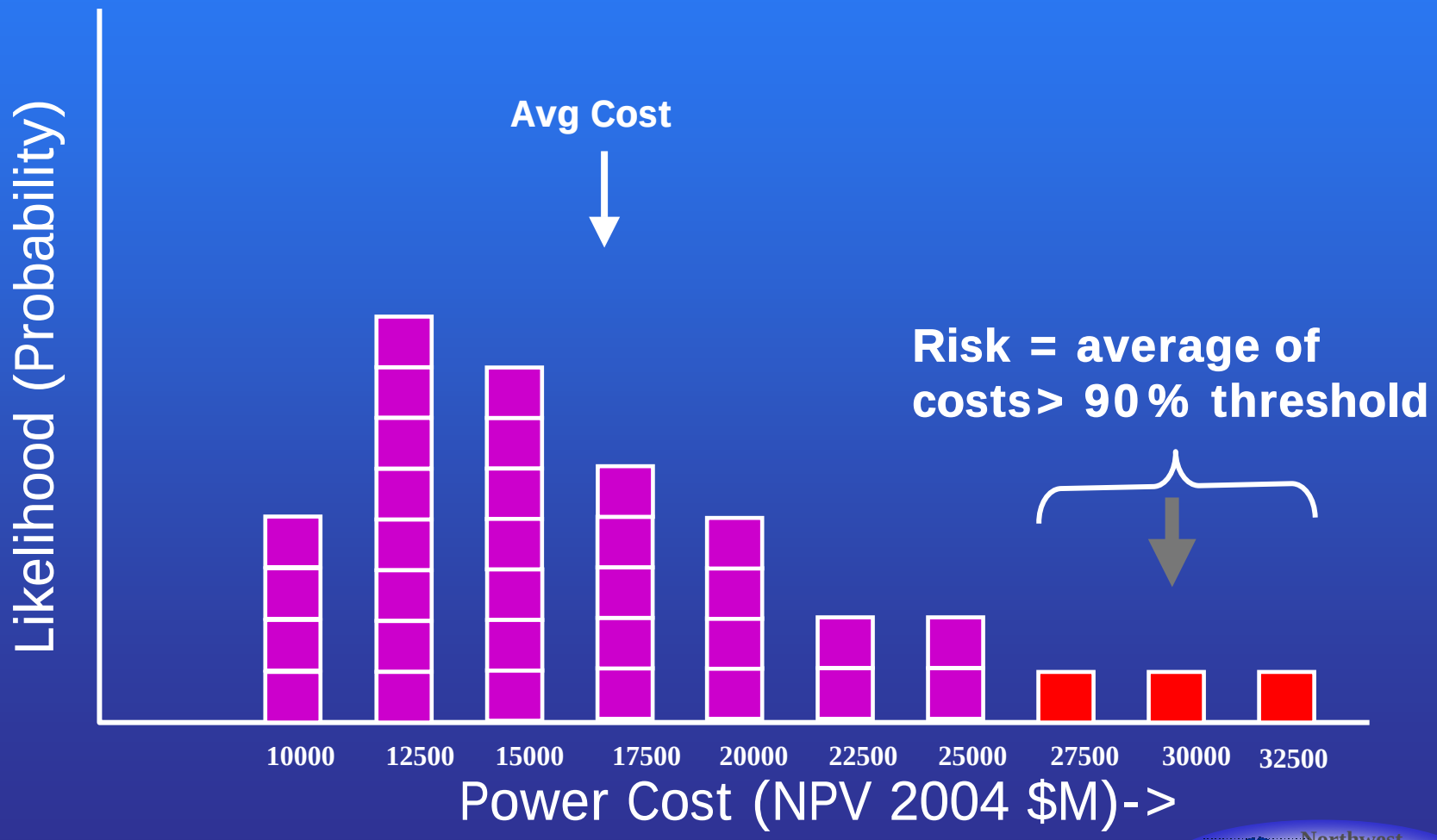
Metrics

- Focus on economic risk
  - Regional planning model
  - More general
  - Relation to coherent measures of risk
- Relate to resource adequacy

Metrics

- Central tendency
  - Gives the decision maker a sense of where the more likely outcomes lie
  - Because it is determined by likelihood only, the median might be better than average or expected value
- Risk
  - Many potential candidates
  - Measure of likelihood and severity of bad outcomes, rather than of predictability
    - » A measure should not penalize a plan because the plan produces less predictable, but strictly better outcomes
    - » It can be less expensive to reduce only the severity and likelihood of bad outcomes

Risk and Expected Cost Associated With A Plan

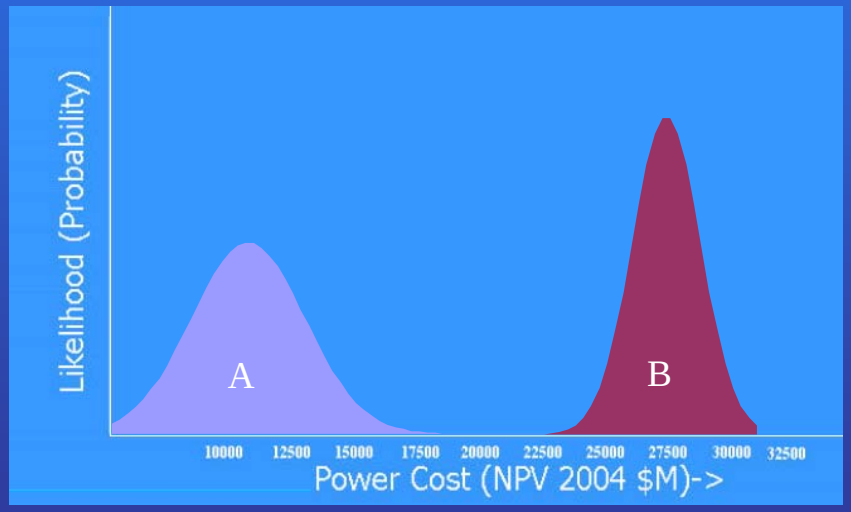


Risk Paradoxes

- Case 1: We choose standard deviation for economic risk measurement.

  Issue: Plan B produces a more predictable outcome, as measured by standard deviation, but all of the outcomes are worse than those associated with Plan A. This risk metric assigns more risk to Plan A than to Plan B.

  Typically, however, a decision maker is looking at cost, too, and could discriminate between these cases.

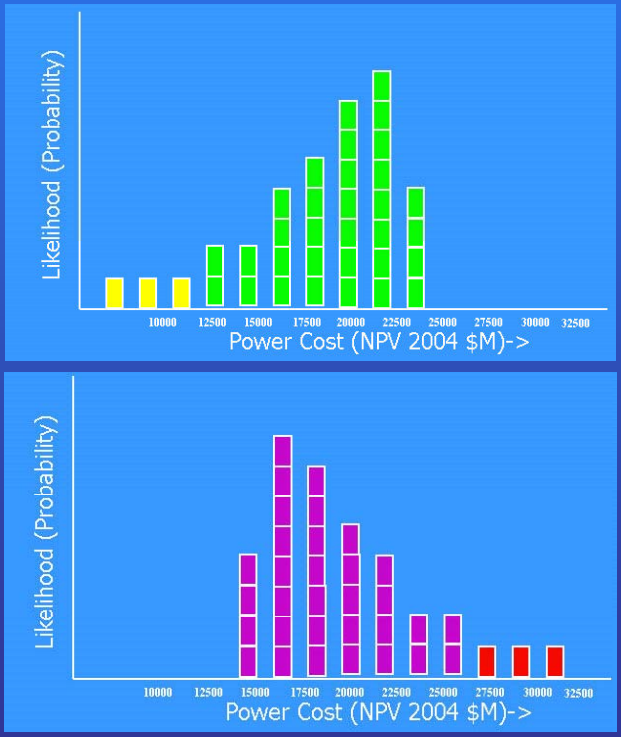


Risk Paradoxes

- Case 2: We choose standard deviation for economic risk measurement.

Issue: Two plans produce quite distinct distributions for cost outcomes. For one of the plans, the outcomes are much worse under certain circumstances than for the other plan.

However, the distributions have identical mean and standard deviation. The risk measure can not discriminate between the plans.



Risk Paradoxes

- Case 3: We choose LOLP for assessing the engineering reliability of two power systems.
  - Issue: We have two systems, both meeting a load of 150MW. The first consists of one 200 MW power plant, forced outage rate (FOR) of 8%. The second system is two 100 MW power plants, FOR also 8%.
  - We know intuitively that portfolio diversity of resources should result in a more reliable system.

Risk Paradoxes

- Case 3: We choose LOLP for assessing the engineering reliability of two power systems.

System 1 (one 200 MW plant)

	Probability		System Generation	Unserved	LOLP
down	0.0800	=FOR_1	0	150	8.00%
up	0.9200	=(1-FOR_1)	200	0	
	1.0000				

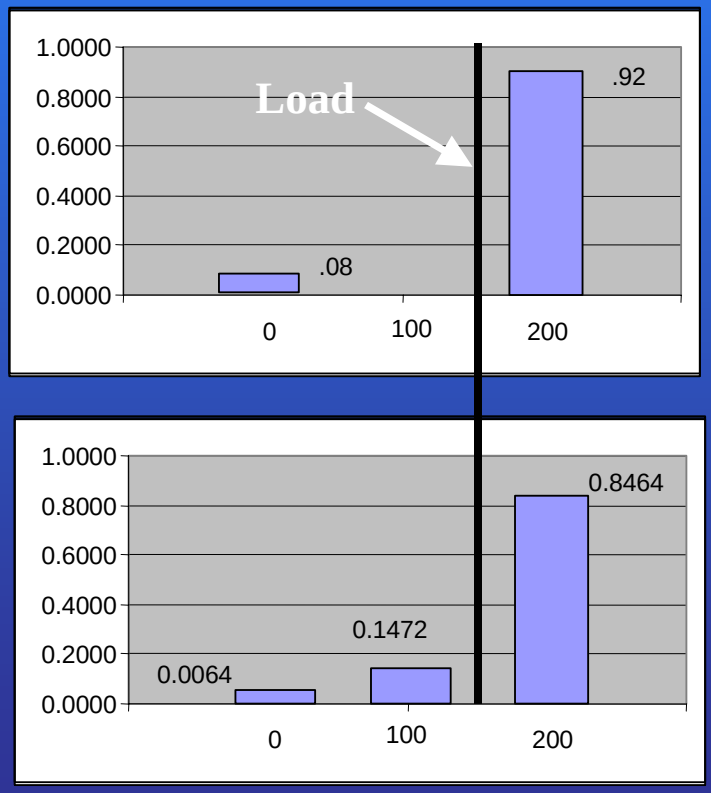
System 2 (two 100 MW plants)

	Probability		System Generation	Unserved	LOLP
(down, down)	0.0064	=FOR_1^2	0	150	
(down, up)	0.0736	=FOR_1*(1-FOR_1)	100	50	15.36%
(up, down)	0.0736	=(1-FOR_1)*FOR_1	100	50	
(up, up)	0.8464	=(1-FOR_1)^2	200	0	
	1.0000				

Risk Paradoxes

- Case 3: We choose LOLP for assessing the engineering reliability of two power systems.

The LOLP of the single unit is lower than that for the diversified system. What is going on here?

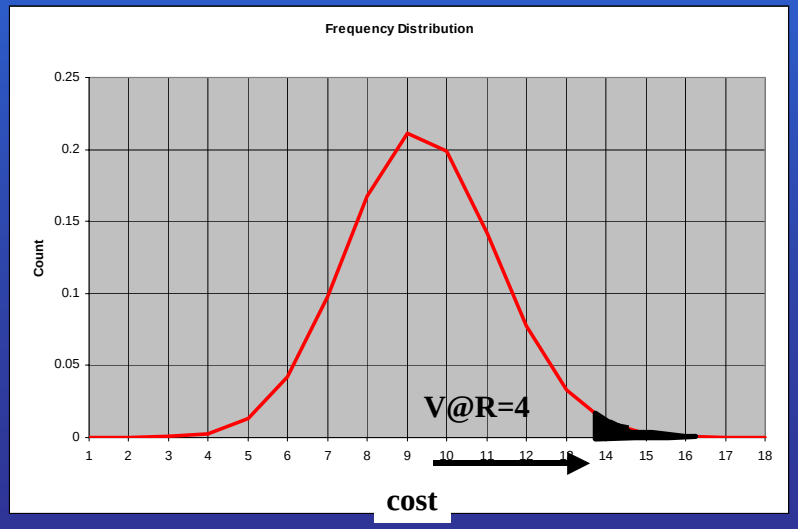


Risk Paradoxes

- Case 4: We choose Value at Risk (VaR) to measure the economic risks associated with merging two power systems.
  - We believe that the diversity of the merged systems should result in less risk.

Risk Paradoxes

- VaR is an estimate of the level of loss on a portfolio which is expected to be equaled or exceeded with a given, small probability.
  - A quantile associated with the "bad tail" of a distribution (e.g., 85th percentile)
  - A time period (e.g., overnight)
  - A reference point (e.g., today's value of the portfolio)



Risk Paradoxes

- Assume a reference point of zero
- Two values of outcome, a loss of 0.00 and a loss of 1.00
- Ten futures

Future	X_1	X_2	X_1+X_2
1	0.00	0.00	0.00
2	0.00	0.00	0.00
3	0.00	0.00	0.00
4	0.00	0.00	0.00
5	0.00	0.00	0.00
6	0.00	0.00	0.00
7	0.00	0.00	0.00
8	0.00	0.00	0.00
9	0.00	1.00	1.00
10	1.00	0.00	1.00
VaR@85%	0.00	0.00	1.00

$$0 = VaR(X_1) + VaR(X_2) < VaR(X_1 + X_2) = 1$$ **!??**

Issues

- These risk measures either ignore or do not adequately reflect the value of the outcomes.
- Diversification means good and bad outcomes offsetting one another. Unless the magnitude of bad outcomes is captured, the value of diversity is not captured.
- If we are concerned about bad outcomes and about diversification, therefore, we need to incorporate more information about the bad outcomes than do quantile measures like LOLP and VaR, or dispersion measures, like standard deviation.

Unserved Energy

System 1 (one 200 MW plant)	Probability	System Generation	Unserved	EUE
down	0.0800	0	150	12.00
up	0.9200	200	0	0.00
	1.0000			12.00

System 2 (two 100 MW plants)	Probability	System Generation	Unserved	EUE
(down, down)	0.0064	0	150	0.96
(down, up)	0.0736	100	50	3.68
(up, down)	0.0736	100	50	3.68
(up, up)	0.8464	200	0	0.00
	1.0000			8.32

Overview

- Issues with risk measures
- Coherent measures of risk
- Relevance to resource adequacy and the metrics the Resource Adeuqacy Technical Committee is discussing

Coherent Measures of Risk

- In 1999, Philippe Artzner, Universite Louis Pasteur, Strasbourg; Freddy Delbaen, Eidgen*f*ossische Technische Hochschule, Zurich; Jean-Marc Eber, Societe Generale, Paris; and David Heath, Carnegie Mellon University, Pittsburgh, Pennsylvania, published *Coherent Measures of Risk* (Math. Finance 9 (1999), no. 3, 203-228) or http://www.math.ethz.ch/~delbaen/ftp/preprints/CoherentMF.pdf
- Addressing problems with VaR
- Developed a system of desirable properties for financial and economic risk measures

Desirable Properties For a Risk Metric ρ

- Subadditivity – For all random outcomes (losses) X and Y,

$$\rho(X+Y) \leq \rho(X)+\rho(Y)$$

- Monotonicity – If $X \leq Y$ for each future, then

$$\rho(X) \leq \rho(Y)$$

- Positive Homogeneity – For all $\lambda \geq 0$ and random outcome X

$$\rho(\lambda X) = \lambda\rho(X)$$

- Translation Invariance – For all random outcomes X and constants α

$$\rho(X+\alpha) = \rho(X) + \alpha$$

The Meaning of Coherence

- *Subadditivity* $\rho(X+Y) \leq \rho(X)+\rho(Y)$: a merger does not create extra risk.
- *Monotonicity* $\rho(X) \leq \rho(Y)$: if all the outcomes associated with a plan are worse, the plan can not be less risky
- *Homogeneity* $\rho(\lambda X) = \lambda\rho(X)$:a limit case of subadditivity, representing what happens when there is precisely no diversification effect.

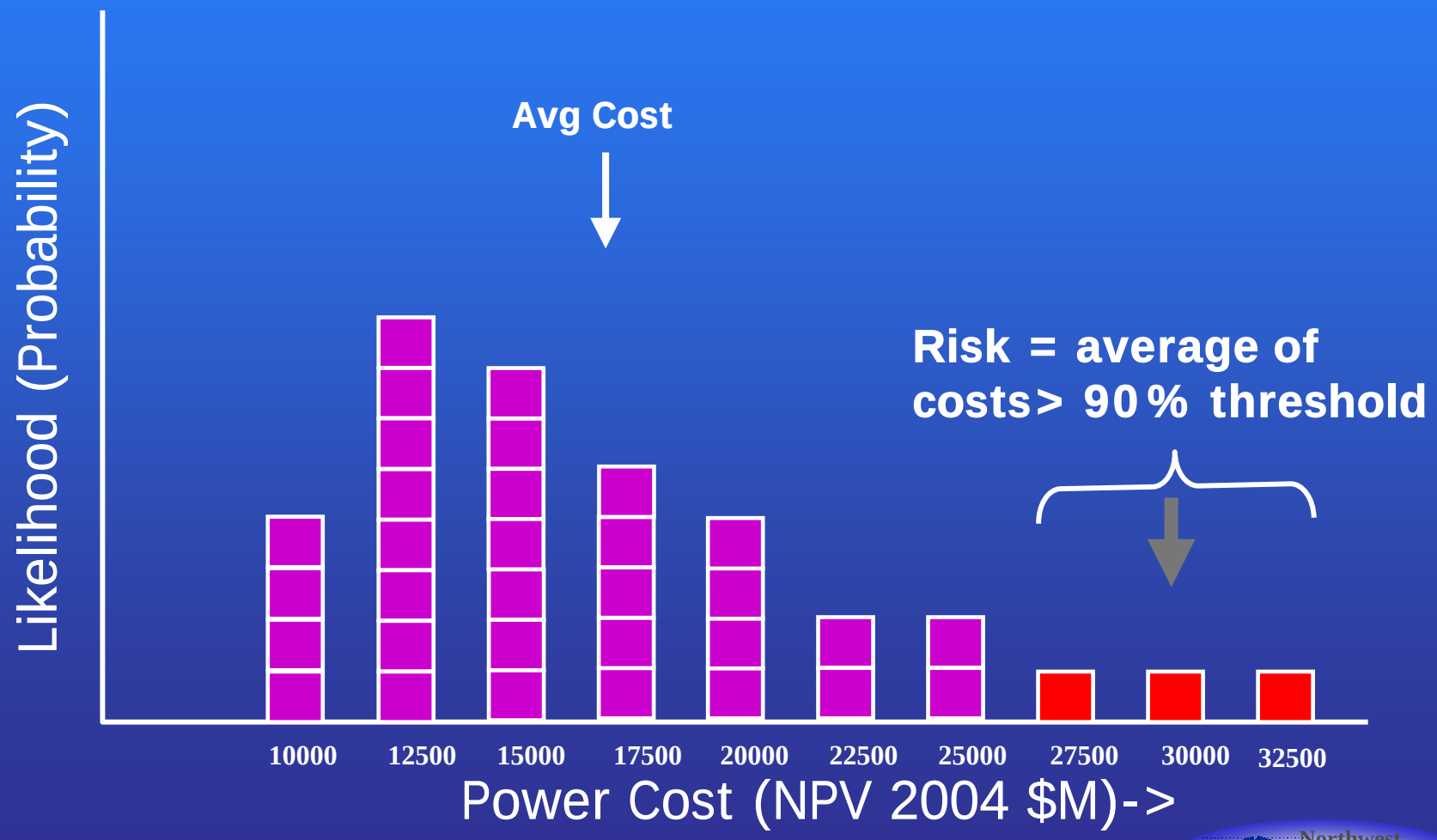
The Meaning of Coherence

- *Translation invariance* $\rho(X+\alpha) = \rho(X) + \alpha$: assures that elements of plan that have *certain* cost properly influence the value of the risk measure.
- Note
  - *Translation invariance* and *Homogeneity* assure that the metric properly rank plans by risk.
  - *Translation invariance* requires that the risk measure have the same units (denomination) as the outcomes.

Risk in the Regional Plan

- The Council considered several alternative coherent measures of risk, such as CVaR, the average of outcomes above some risk threshold
- They settled on TailVaR_{90}, in part because it is easier to explain and can be used to make statements about probabilities

Risk and Expected Cost Associated With A Plan



Importance of Multiple Perspectives on Risk

- TailVaR90
- CVaR20000
- Standard Deviation
- VaR90
- 90th Quintile
- Loss of Load Probability (LOLP)
- Resource - Load Balance
- Incremental Cost Variation
- Average Power Cost Variation (Rate Impact)
- Maximum Incremental Cost Increase
- Exposure to Wholesale Market Prices
- Imports and Exports

Overview

- Issues with risk measures
- Coherent measures of risk
- Relevance to resource adequacy and the metrics the Resource Adequacy Technical Committee is discussing

Comparing Risk Measures

How does all this relate to the work or the RATC?

- LOLP, unserved energy, and reserve margin *are* indirect economic risk measures
  - Develop these by considering costs of not meeting load
- They do not, however, the cost of *meeting* load.

Advantages of Regional Economic Risk Measures

- More general
- More sensitive to imminent inadequacy
  - Plans produced by the regional model passed the LOLP and UE tests
- Reflect how utilities actually plan, i.e., incorporate economics
- Lead to resource choices that do not *increase* risks *other than* failure to meet load
  - e.g., SCCTs can increase natural gas price risk

Disadvantages of Regional Economic Risk Measures

- Require more information and effort to compute
- Requires information that tends to be commercially sensitive, such as contract information
- Lead to resource choices
  - If utilities do not like to be told *how much* to build, they will dislike being told *what* to build
- Lack of regional consensus on what economic risk means, how to model it, and how to measure it

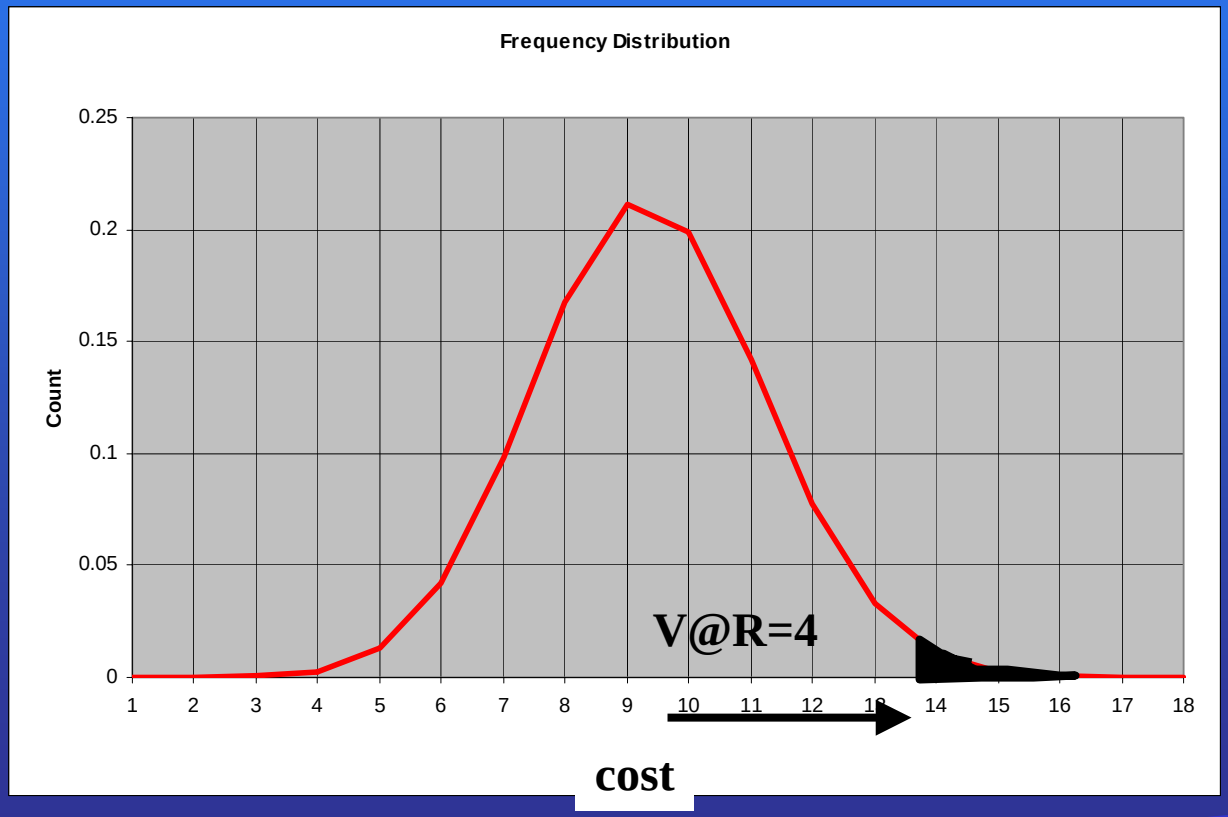
Conclusions

- Economic risk measures are better than LOLP, UE, and reserve margin measures for individual utilities, but not (yet) practical for the region
- Michael's preference is UE > LOLP > reserve margin
  - UE captures diversity
  - Reserve margin is only a rule-of-thumb for obtaining a given level of UE, LOLP, or economic performance

End

V@R

95% one-day V@R



Northwest Power and Conservation Council
Metrics

A List of Loss Scenarios

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

Define a measure of risk $\rho(X) = \text{Maximum}\{X_i\}$

Northwest Power and Conservation Council
Metrics

Subadditivity

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

$$\rho(X+Y) \leq \rho(X)+\rho(Y)$$

Northwest Power and Conservation Council
Metrics

Monotonicity

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

If $X \leq Y$ for each scenario, then $\rho(X) \leq \rho(Y)$

Northwest Power and Conservation Council
Metrics

Positive Homogeneity

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

For all $\lambda \geq 0$ and random loss X, $\rho(\lambda X) = \lambda\rho(Y)$

Northwest Power and Conservation Council
Metrics

Translation Invariance

Scenario	X_1	X_2	X_1+X_2	$X_3 = 2*X_1$	$X_4 = X_1+1$
1	1.00	0.00	1.00	2.00	2.00
2	2.00	0.00	2.00	4.00	3.00
3	3.00	0.00	3.00	6.00	4.00
4	4.00	1.00	5.00	8.00	5.00
5	3.00	2.00	5.00	6.00	4.00
6	2.00	3.00	5.00	4.00	3.00
7	1.00	4.00	5.00	2.00	2.00
8	0.00	3.00	3.00	0.00	1.00
9	0.00	2.00	2.00	0.00	1.00
10	0.00	1.00	1.00	0.00	1.00
Maximum Loss	4.00	4.00	5.00	8.00	5.00

For all random losses X and constants α

$$\rho(X+\alpha) = \rho(X) + \alpha$$

Northwest Power and Conservation Council Metrics

Standard Deviation violates monotonicity

Scenario	X_1	X_2
1	1.00	5.00
2	2.00	5.00
3	3.00	5.00
4	4.00	5.00
5	5.00	5.00
6	5.00	5.00
7	4.00	5.00
8	3.00	5.00
9	2.00	5.00
10	1.00	5.00
$E[Loss]$	3.00	5.00
$StDev[Loss]$	1.41	0.00
$E[Loss]+2*StDev[Loss]$	5.83	5.00

Northwest Power and Conservation Council
Metrics

Metric space and Distance

- $d(p,q)$ is the "distance" between elements p and q in a set if
  - $d(p,q)>0$
  - $d(p,q)=d(q,p)$
  - $d(p,q)\leq d(p,r)+d(r,q)$, for all r

Northwest Power and Conservation Council

Normed space and Norm

- $\| . \|$ is the "norm" on an element p in a set if
  - $\|\alpha p\| = |\alpha| \, \|p\|$
  - $\|p+q\| \leq \|p\| + \|q\|$, for all q
- Note: for this to make sense, p must have value in the complex numbers

Northwest Power and Conservation Council